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Research Article

Python Approach For Solving Fractional Order Modified KdV Burger's Equation By Adomian Decomposition Method

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Abstract. The present work is devoted to solve fractional order modified KdV Burger's equation. We consider a time fractional order modified KDV Burger's equation in the Caputo sense. Adomian decomposition method is employed to find the approximate solution of this equation. We developed Python program for Adomian Decomposition Method to find numerical solutions. We obtained the approximate solutions of time fractional order modified KDV Burger's equation by considering higher nonlinearity. The obtained results are compared with exact solution. The Adomian Decomposition Method provides an approach for approximating the solution of highly non-linear equations of fractional order.

Keywords. Fractional order differential equation, Caputo derivative, ADM, KDV Burger equation, Python

Mathematics Subject Classification (2020). 26A33, 35R11, 35Q53, 65N35, 65Y20

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1. Introduction

Fractional-order differential equations (FODEs) are widely used to model long-memory processes, but their analysis is more complex than the classical integer-order equations. Consequently, numerical methods such as the *Adomian Decomposition Method* (ADM) (Kulkarni *et al*. [10]; Li and Pang [11]), *Homotopy Perturbation Method* (HPM), *Laplace ADM* (LADM), and *Homotopy Analysis Method* (HAM) are commonly employed to obtain approximate solutions. Among these, ADM is particularly effective for nonlinear FODEs (Zeng *et al*. [17]).

The *Korteweg-de Vries* (KdV) equation, describes soliton-like waves of small amplitude and finds applications in fluid mechanics and nonlinear lattices. The Burgers equation models viscous fluid dynamics and shock wave propagation, balancing nonlinear advection with diffusion. Later researchers derived exact solutions and explored its connection to turbulence and shock theory (see, Nieto and Rodríguez-López [12]; Panwar *et al*. [13]; Wei *et al*. [15]).

The *KdV-Burgers* (KdVB) equation combines effects of dispersion, dissipation, and nonlinearity, modeling wave propagation in fluid-filled elastic or viscoelastic tubes (Khuri [9]). Travelling wave solutions and other analytical forms have been extensively studied using various techniques. The KdVB equation is also useful in describing wave dynamics in fluids, elastic tubes, and other applied physics systems (see, Alderremy *et al*. [1]; Alqahtani and Hagag [2]; Atta and Youssri [3]; Deng and Deng [5]; Ganie *et al*. [6]; Hashemi *et al*. [7]; Heydari *et al*. [8]). The present article is devoted to construct the approximate solutions for highly nonlinear modified KDVB equation.

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u^n \frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0, \quad 0 < \alpha \le 1, n = 1,2,3,4, \tag{1}$$

obtained by ADM.

1.1 Basic Definitions

Definition 1.1 ([12, 13])**.** Caputo derivative with fractional order $\alpha > 0$ is given by

$$D^\alpha h(t) = \frac{1}{\Gamma(l-\alpha)} \int_a^t \frac{h^{(l)}(\tau)}{(t-\tau)^{\alpha+1-l}} d\tau, \quad l-1 < Re(\alpha) \le l, l \in \mathbb{N}, \tag{2}$$

where α is positive real number and function h initially takes value as a.

Definition 1.2 ([12, 13])**.** For integer $l \ge \alpha$, Caputo derivative operator with time fractional order $\alpha > 0$ is defined as

$$D_t^\alpha h(y,t) = \frac{\partial^\alpha h(y,t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(l-\alpha)} \int_0^t (t-\tau)^{l-\alpha-1} \frac{\partial^l h(y,\tau)}{\partial \tau^l} d\tau, & \text{if } l-1 < \alpha < l, \\ \frac{\partial^l h(y,t)}{\partial t^l}, & \text{if } \alpha = l \in \mathbb{N}. \end{cases} \tag{3}$$

Definition 1.3 ([12, 13])**.** The Riemann Liouville integral operator with fractional order $\alpha \ge 0$ for a function $h \in C_p$, $p \ge -1$ is defined as

$$J^\alpha h(y) = \frac{1}{\Gamma(\alpha)} \int_0^y (y-t)^{\alpha-1} h(t) dt, \quad \alpha > 0, y > 0. \tag{4}$$

Following are some properties:

(i) for $h \in C_p$, $p \ge -1$, $\alpha, \beta \ge 0$, $\delta \ge -1$,

(ii) $J^0 h(y) = h(y)$, $J^\alpha y^\delta = \frac{\Gamma(\delta+1)}{\Gamma(\alpha+\delta+1)} y^{\alpha+\delta}$,

(iii) $J^\alpha J^\beta h(y) = J^{\alpha+\beta} h(y)$, $J^\alpha J^\beta h(y) = J^\beta J^\alpha h(y)$,

(iv) if $l-1 < \alpha \le l$, $l \in \mathbb{N}$ and $h \in C_p^l$, $p \ge -1$, then

(a) $D^{\alpha}J^{\alpha}h(y) = h(y)$,

(b) $J^{\alpha}D^{\alpha}h(y) = h(y) - \sum_{q=0}^{l-1} h^{q}(0^{+})\frac{y^{q}}{q!}$, $y > 0$.

1.2 The Fractional Adomian Decomposition Method

We know that the standard form of equation (1) in an operator form is given by

$$D_t^{\alpha}u + u^n\frac{\partial u}{\partial x} + L_x u = 0, \quad 0 < \alpha \le 1, t > 0, \tag{5}$$

where $L_x u = -a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3}$.

After applying the operator J^{α} on both sides of the above equations, we get

$$u(x,t) = h(x) - J^{\alpha}\phi(u) - J^{\alpha}L_x u,$$

where $\phi(u) = u^n u_x$.

In decomposition method, we assume that the solution of $u(x,t)$ is given in the series form by

$$u(x,t) = \sum_{q=0}^{\infty} u_q(x,t),$$

where the term $\phi(u)$ is nonlinear and decomposes into the infinite series of polynomial as follows

$$\phi(u) = \sum_{q=0}^{\infty} A_q(u_0, u_1, \ldots, u_q),$$

where A_q represents Adomian Polynomials and are calculated with the formula

$$A_q = \frac{1}{q!}\left[\frac{d^q}{d\lambda^q}\phi\left(\sum_{q=0}^{\infty}\lambda^q u_q\right)\right]_{\lambda=0} = \frac{1}{q!}\left[\frac{d^q}{d\lambda^q}\left(\left(\sum_{q=0}^{\infty}\lambda^q u_q(x,t)\right)\left(\sum_{q=0}^{\infty}\lambda^q u_q(x,t)\right)_x\right)\right].$$

After substituting all the series, equation (6) will become

$$\sum_{q=0}^{\infty} u_q(x,t) = h(x) - J^{\alpha}\left(\sum_{q=0}^{\infty} A_q\right) - J^{\alpha}L_x\left(\sum_{q=0}^{\infty} u_q(x,t)\right).$$

The recurrence relation can be developed as follows

$$u_0 = u(x,0) = h(x),$$
$$u_{q+1} = -J^{\alpha}(A_q) - J^{\alpha}L_x(u_q), \quad q = 0,1,2,\ldots .$$

Hence, the approximate solution will take the form

$$u = u_0 + u_1 + u_2 + u_3 + \ldots .$$

1.3 Uniqueness and Convergence

Theorem 1.1. *Time-fractional partial differential equation is given by*

$$D^{\alpha}u + Lu + Nu = 0, \quad r - 1 < \alpha \le r, \tag{6}$$

where $r = \lceil\alpha\rceil \in \mathbb{N}$ *and* $D = \frac{\partial}{\partial x}$ *with* $z(x,0) = z_0$ *has a unique solution for* $0 < \frac{kt^{\alpha}}{\Gamma(\alpha+1)} < 1$, *for some constant* k.

Proof. Let $I = [0,T]$ be an interval and $X = (C(I), \|\cdot\|)$ is Banach space of all continuous functions. Define norm $\|u(t)\| = \max_{t\in I}|u(t)|$. Consider a function $S : X \to X$, such that $S(u(t)) = u_0 - J^{\alpha}(Lu) - J^{\alpha}(Nu)$. If the nonlinear term Nu is Lipschitzian, then we have

$$|N(u) - N(f)| \le k_1|u - f|,$$

where $f \in X$ and k_1 is Lipschitz constant. Let $u_1, u_2 \in X$. Consider

$$\begin{aligned}
\|S(u_1)-S(u_2)\| &= \max_{t\in I}|-J^\alpha N(u_1(t))-J^\alpha L(u_1(t))+J^\alpha N(u_2(t))+J^\alpha L(u_2(t))| \\
&= \max_{t\in I}|-J^\alpha(Nu_1-Nu_2)-J^\alpha(Lu_1-Lu_2)| \\
&= \max_{t\in I}|J^\alpha(Nu_1-Nu_2)+J^\alpha(Lu_1-Lu_2)| \\
&\le \max_{t\in I}|J^\alpha(Nu_1-Nu_2)|+|J^\alpha(Lu_1-Lu_2)| \\
&\le \max_{t\in I}|J^\alpha(Nu_1-Nu_2)|+\max_{t\in I}|J^\alpha(Lu_1-Lu_2)|.
\end{aligned}$$

Also, if $L(u(t))$ is Lipschitzian, then we have $|L(u)-L(f)| \le k_2|u-f|$, where k_2 is Lipschitz constant. Therefore,

$$\begin{aligned}
\|S(u_1)-S(u_2)\| &\le \max_{t\in I}(k_1J^\alpha|u_1-u_2|+k_2J^\alpha|u_1-u_2|) \\
&\le (k_1+k_2)\|u_1-u_2\|\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\le k\|u_1-u_2\|,
\end{aligned}$$

where $k=\frac{(k_1+k_2)t^\alpha}{\Gamma(\alpha+1)}$.

Therefore, whenever $0<k<1$, S is a contraction mapping. Hence by the Banach fixed point theorem, the time-fractional partial differential equation (10) has a unique solution. □

Convergence can be proved using following theorem.

Theorem 1.2. *Let $S_n=\sum_{i=0}^{n-1}u_i(x,t)$ be the nth partial sum then sequence $\{S_n\}$ is a Cauchy sequence in Banach space X.*

Proof. Using the recurrence relation for $u(x,t)$ given by

$$u_0=\sum_{k=0}^{r-1}u^{(k)}(0^+)\frac{x^k}{k!},$$

$$u_{n+1}=-J^\alpha[Lu_n+A_n], \quad \text{for } n=0,1,2,\ldots,$$

where A_n's are Adomian polynomials.
For $m\in\mathbb{N}$, consider

$$\begin{aligned}
\|S_{n+m}-S_n\| &= \max_{t\in I}|S_{n+m}-S_n| \\
&= \max_{t\in I}\left|\sum_{i=n}^{n+m-1}u_i(x,t)\right| \\
&= \max_{t\in I}\left|-J^\alpha\sum_{i=n}^{n+m-1}Lu_{i-1}(x,t)-J^\alpha\sum_{i=n}^{n+m-1}Nu_{i-1}(x,t)\right| \\
&= \max_{t\in I}|J^\alpha(LS_{n+m-1}-LS_{n-1})+J^\alpha(NS_{n+m-1}-NS_{n-1})| \\
&\le \max_{t\in I}J^\alpha|LS_{n+m-1}-LS_{n-1}|+\max_{t\in I}J^\alpha|NS_{n+m-1}-NS_{n-1}| \\
&\le k_2\max_{t\in I}J^\alpha|S_{n+m-1}-S_{n-1}|+k_1\max_{t\in I}J^\alpha|S_{n+m-1}-S_{n-1}| \\
&\le (k_1+k_2)\frac{t^\alpha}{\Gamma(\alpha+1)}\|S_{n+m-1}-S_{n-1}\| \\
&\le k\|S_{n+m-1}-S_{n-1}\|,
\end{aligned}$$

where $k = \frac{(k_1+k_2)t^\alpha}{\Gamma(\alpha+1)}$. In a similar way, we have

$$\begin{aligned}\|S_{n+m} - S_n\| &\le k^2\|S_{n+m-2} - S_{n-2}\| \le \cdots \le k^n\|S_m - S_0\| \le k^n\|S_1 - S_0\|, \quad \text{for } m = 1\\ &\le K^n\|u_1\|.\end{aligned}$$

In particular, for $p > q$, for $p, q \in N$. Consider

$$\begin{aligned}\|S_p - S_q\| &\le \|S_{q+1} - S_q\| + \|S_{q+2} - S_{q+1}\| + \cdots + \|S_p - S_{p-1}\|\\ &\le (k^q + k^{q+1} + \cdots + k^{p-1})\|u_1\|\\ &\le k^q(1 + k + k^2 + \cdots + k^{p-q-1})\|u_1\|\\ &\le k^q\left[\frac{1-k^{p-q}}{1-k}\right]\|u_1\|.\end{aligned}$$

Since, $0 < k < 1$, therefore $1 - k^{p-q} < 1$, so we obtain, $\|S_p - S_q\| \le \frac{k^q}{1-k}\|u_1\|$.

As $u(t)$ is bounded, $\|u_1\| < \infty$. So we have $\lim_{p\to\infty}\|S_p - S_q\| \to 0$. Therefore, $\{S_n\}$ is a Cauchy sequence in X. Hence, the solution of the given equation is convergent. □

2. Numerical Example

We consider mKdV Burger's equation with time-fractional order as follows

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u^n\frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0, \quad 0 < \alpha \le 1 \tag{7}$$

with the initial condition,

$$u(x,0) = \frac{3}{25}\frac{a^2}{b}\operatorname{sech}\left(\frac{a}{10b}x\right)^2 + \frac{6}{25}\frac{a^2}{b}\left[1 - \tanh\left(\frac{a}{10b}x\right)\right]. \tag{8}$$

According to the Adomian Decomposition Method, we have

$$\begin{aligned}u_0 &= u(x,0) = v_0,\\ v_1 &= -J^\alpha(A_0 - a(u_0)_{xx} + b(u_0)_{xxx}),\\ v_2 &= -J^\alpha(A_1 - a(u_1)_{xx} + b(u_1)_{xxx}),\\ v_3 &= -J^\alpha(A_2 - a(u_2)_{xx} + b(u_2)_{xxx}),\\ &\vdots\end{aligned}$$

Case 1: $n = 1$

The mKdV Burger's equation (1) will take the form as

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u\frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0, \quad 0 < \alpha \le 1. \tag{9}$$

An approximate series solution is as follows

$$\begin{aligned}u(x,t) &= u_0 + u_1 + u_2 + \ldots\\ &= \left\{\frac{3}{25}\frac{a^2}{b}\operatorname{sech}^2\left(\frac{a}{10b}x\right) + \frac{6}{25}\frac{a^2}{b}\left[1 - \tanh\left(\frac{a}{10b}x\right)\right]\right\}\\ &\quad + \left\{\frac{18\left(t^\alpha\cosh\left(\frac{1}{10}x\right) + t^\alpha\sinh\left(\frac{1}{10}x\right)\right)}{3125\cosh^3\left(\frac{1}{10}x\right)\Gamma(\alpha+1)}\right\}\\ &\quad + \left\{\frac{54\left(2t^{2\alpha}\cosh^2\left(\frac{1}{10}x\right) + 2t^{2\alpha}\cosh\left(\frac{1}{10}x\right)\sinh\left(\frac{1}{10}x\right) - 3t^{2\ \alpha}\right)}{390625\cosh^4\left(\frac{1}{10}x\right)\Gamma(2\alpha+1)}\right\} + \ldots .\end{aligned} \tag{10}$$

The observed kink type of traveling wave solution and its graphical representation for *Case* 1 presented by Pawar *et al*. [13] serves as the foundation for our study. Here, we generalize their approach to treat for $n = 2,3$ and 4.

Case 2: $n = 2$

The mKdV Burger's equation will (1) take the form as

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u^2\frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0, \quad 0 < \alpha \le 1. \tag{11}$$

Adomian polynomials for nonlinear term $u^2\frac{\partial u}{\partial x}$ are given by

$$\begin{aligned}
A_0 &= u_0^2 u_{0x},\\
A_1 &= 2u_0u_1u_{0x} + u_0^2u_{1x},\\
A_2 &= 2u_0u_2u_{0x} + u_1^2u_{0x} + 2u_0u_1u_{1x} + u_0^2u_{2x},\\
A_3 &= 2u_0u_3u_{0x} + 2u_1u_2u_{0x} + 2u_0u_2u_{1x} + u_1^2u_{1x} + 2u_0u_1u_{2x} + u_0^2u_{3x},\\
&\vdots
\end{aligned}$$

We obtain the solution as

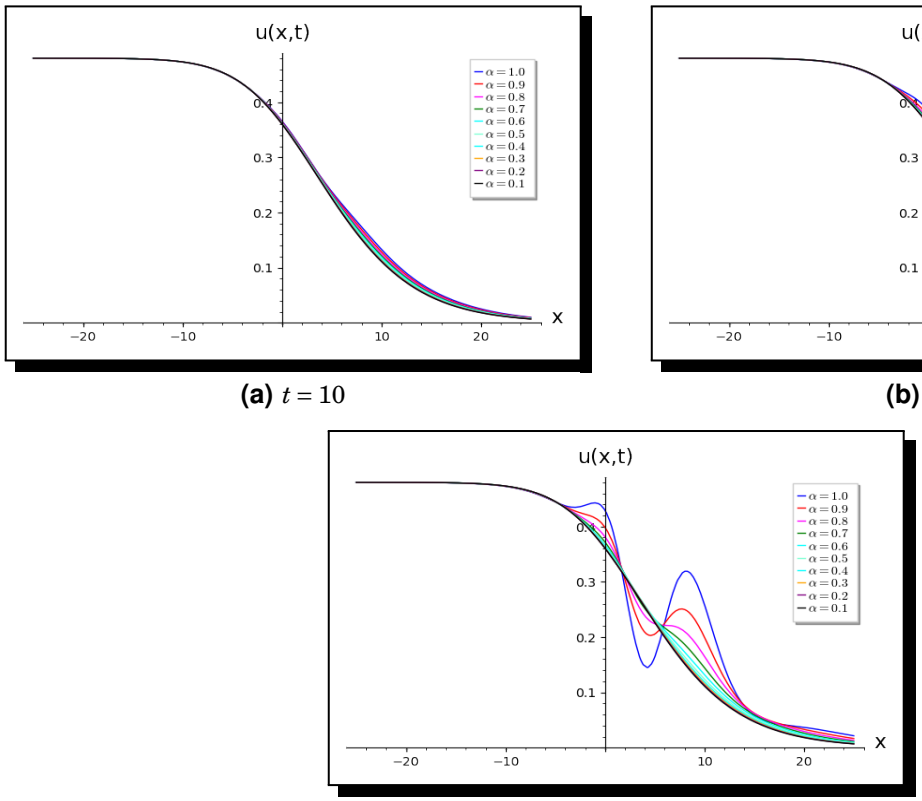
$$\begin{aligned}
v_0 &= \frac{3}{25}\frac{a^2}{b}\operatorname{sech}^2\left(\frac{a}{10b}x\right) + \frac{6}{25}\frac{a^2}{b}\left[1 - \tanh\left(\frac{a}{10b}x\right)\right],\\
v_1 &= \frac{9}{78125\cosh^7\left(\frac{x}{10}\right)\Gamma(\alpha+1)}\left(50t^\alpha\cosh^5\left(\frac{x}{10}\right) - 63t^\alpha\cosh^3\left(\frac{x}{10}\right) + 15t^\alpha\cosh\left(\frac{x}{10}\right)\right.\\
&\quad \left. + \left(50t^\alpha\cosh^4\left(\frac{x}{10}\right) - 37t^\alpha\cosh^2\left(\frac{x}{10}\right) + 3t^\alpha\right)\sinh\left(\frac{x}{10}\right)\right),\\
v_2 &= \frac{9}{976562500\cosh^{12}\left(\frac{x}{10}\right)\Gamma(2\alpha+1)}\left(30000t^{2\alpha}\cosh^{10}\left(\frac{x}{10}\right) - 230024t^{2\alpha}\cosh^8\left(\frac{x}{10}\right)\right.\\
&\quad + 412378t^{2\alpha}\cosh^6\left(\frac{x}{10}\right) - 242049t^{2\alpha}\cosh^4\left(\frac{x}{10}\right) + 35154t^{2\alpha}\cosh^2\left(\frac{x}{10}\right)\\
&\quad + 2\left(15000t^{2\alpha}\cosh^9\left(\frac{x}{10}\right) - 107488t^{2\alpha}\cosh^7\left(\frac{x}{10}\right) + 154317t^{2\alpha}\cosh^5\left(\frac{x}{10}\right)\right.\\
&\quad \left.\left. - 56328t^{2\alpha}\cosh^3\left(\frac{x}{10}\right) + 2430t^{2\alpha}\cosh\left(\frac{x}{10}\right)\right)\sinh\left(\frac{x}{10}\right) - 594t^{2\alpha}\right).
\end{aligned}$$

We obtain an approximate series solution as follows

$$\begin{aligned}
u(x,t) &= \left\{\frac{3}{25}\frac{a^2}{b}\operatorname{sech}^2\left(\frac{a}{10b}x\right) + \frac{6}{25}\frac{a^2}{b}\left[1 - \tanh\left(\frac{a}{10b}x\right)\right]\right\}\\
&\quad + \left\{\frac{9}{78125\cosh^7\left(\frac{1}{10}x\right)\Gamma(\alpha+1)}\left(50t^\alpha\cosh^5\left(\frac{1}{10}x\right) - 63t^\alpha\cosh^3\left(\frac{1}{10}x\right)\right.\right.\\
&\quad \left.\left. + 15t^\alpha\cosh\left(\frac{1}{10}x\right) + \left(50t^\alpha\cosh^4\left(\frac{1}{10}x\right) - 37t^\alpha\cosh^2\left(\frac{1}{10}x\right) + 3t^\alpha\right)\sinh\left(\frac{1}{10}x\right)\right)\right\}\\
&\quad + \left\{\frac{9}{976562500\cosh^{12}\left(\frac{1}{10}x\right)\Gamma(2\alpha+1)}\left(30000t^{2\alpha}\cosh^{10}\left(\frac{1}{10}x\right) - 230024t^{2\alpha}\cosh^8\left(\frac{1}{10}x\right)\right.\right.\\
&\quad \left.\left. + 412378t^{2\alpha}\cosh^6\left(\frac{1}{10}x\right) - 242049t^{2\alpha}\cosh^4\left(\frac{1}{10}x\right) + 35154t^{2\alpha}\cosh^2\left(\frac{1}{10}x\right)\right.\right.
\end{aligned}$$

$$+2\left(15000t^{2\alpha}\cosh^9\left(\frac{1}{10}x\right)-107488t^{2\alpha}\cosh^7\left(\frac{1}{10}x\right)+154317t^{2\alpha}\cosh^5\left(\frac{1}{10}x\right)\right.$$
$$\left.-56328t^{2\alpha}\cosh^3\left(\frac{1}{10}x\right)+2430t^{2\alpha}\cosh\left(\frac{1}{10}x\right)\right)\sinh\left(\frac{1}{10}x\right)-594t^{2\alpha}\Big)\Big\}+\dots\,.$$

From Figure 1, we observe the kink type travelling wave solution for $0<\alpha\le 1$, $t=10$ and the variation in wave patterns for $0.9\le\alpha\le 1$, $t=20$ and $t=30$.

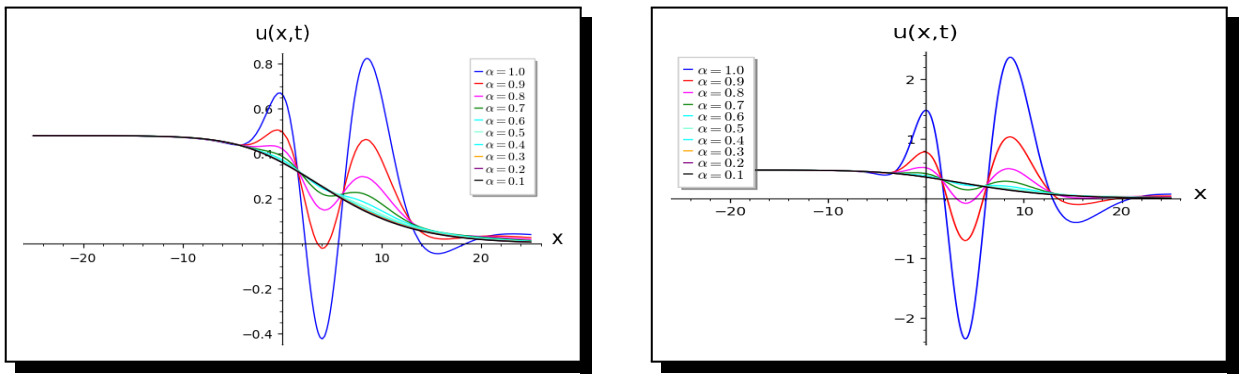


(a) $t=10$ **(b)** $t=20$

(c) $t=30$

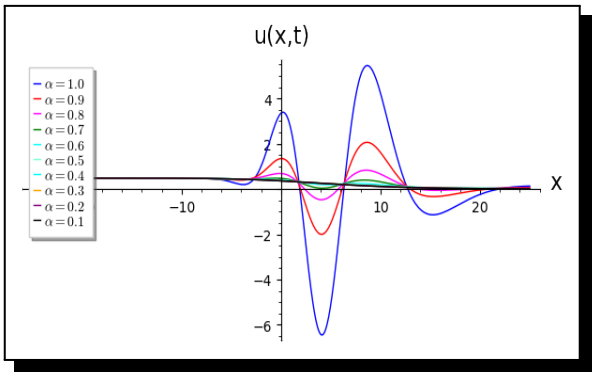
Figure 1. Solution behavior across different $t\le 30$

In Figure 2, we observe that the behavior of the traveling waves follows a specific pattern as time increases in the range $50\le t\le 100$.



(a) $t=50$ **(b)** $t=75$

Figure 2 (*continued*)



(c) $t = 100$

Figure 2. Solution behavior across different $t > 30$

Case 3: $n = 3$

The mKdV Burger's equation (1) will take the form as

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u^3\frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0, \quad 0 < \alpha \le 1. \tag{12}$$

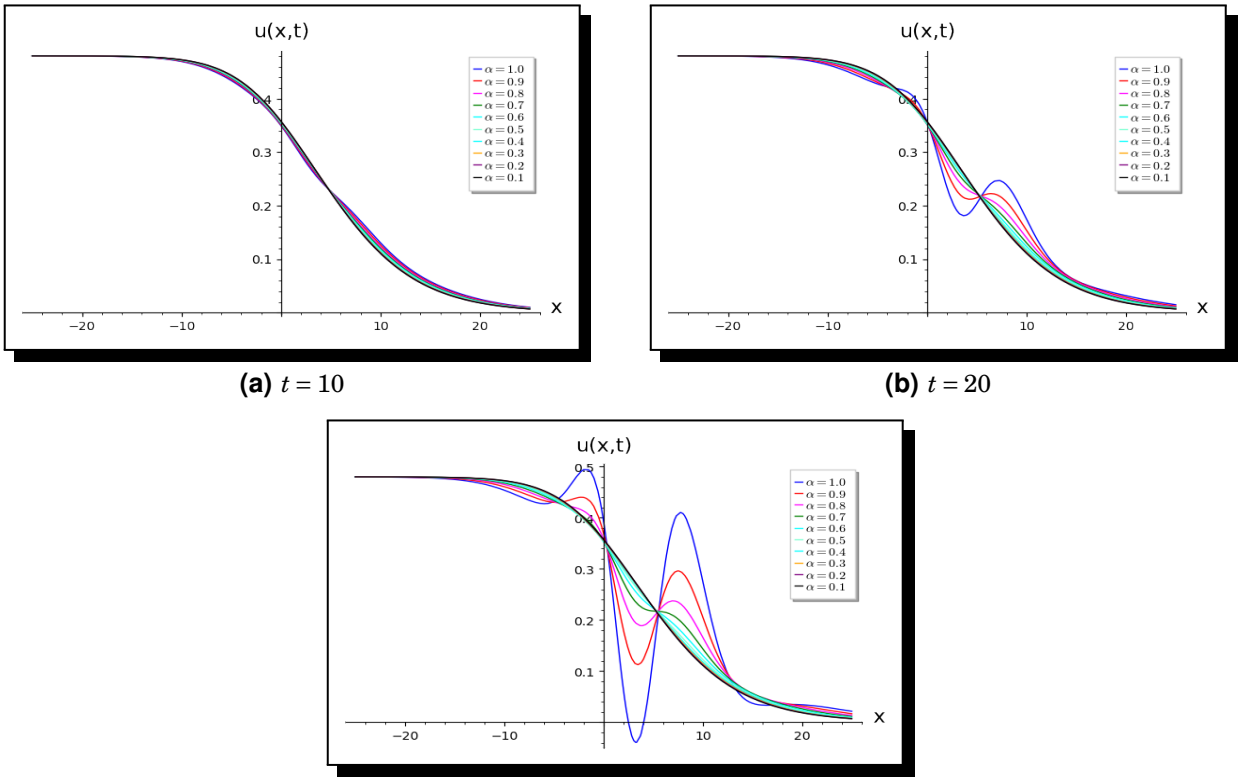
Adomian polynomials for nonlinear term $u^3\frac{\partial u}{\partial x}$ are given by

$$\begin{aligned}
A_0 &= u_0^3 u_{0x},\\
A_1 &= 3u_0^2 u_{0x}u_1 + u_0^3 u_{1x},\\
A_2 &= 3u_0 u_{0x}u_1^2 + 3u_0^2 u_1 u_{1x} + 3u_0^2 u_{0x}u_2 + u_0^3 u_{2x},\\
A_3 &= u_{0x}u_1^3 + 3u_0 u_1^2 u_{1x} + 6u_0 u_{0x}u_1 u_2 + 3u_0^2 u_{1x}u_2 + 3u_0^2 u_1 u_{2x} + 3u_0^2 u_{0x}u_3 + u_0^3 u_{3x},\\
&\vdots
\end{aligned}$$

We obtain the solution as

$$\begin{aligned}
v_0 &= \frac{3}{25}\frac{a^2}{b}\operatorname{sech}^2\left(\frac{a}{10b}x\right) + \frac{6}{25}\frac{a^2}{b}\left[1 - \tanh\left(\frac{a}{10b}x\right)\right],\\
v_1 &= \frac{9}{1953125\cosh^9\left(\frac{x}{10}\right)\Gamma(\alpha+1)}\left(1250t^\alpha\cosh^7\left(\frac{x}{10}\right) - 1731t^\alpha\cosh^5\left(\frac{x}{10}\right) + 36t^\alpha\cosh^3\left(\frac{x}{10}\right)\right.\\
&\quad \left. + 63t^\alpha\cosh\left(\frac{x}{10}\right) + \left(1250t^\alpha\cosh^6\left(\frac{x}{10}\right) - 769t^\alpha\cosh^4\left(\frac{x}{10}\right) - 108t^\alpha\cosh^2\left(\frac{x}{10}\right) + 9t^\alpha\right)\sinh\left(\frac{x}{10}\right)\right),\\
&\vdots\\
u(x,t) &= \left\{\frac{3}{25}\frac{a^2}{b}\operatorname{sech}^2\left(\frac{a}{10b}x\right) + \frac{6}{25}\frac{a^2}{b}\left[1 - \tanh\left(\frac{a}{10b}x\right)\right]\right\} + \frac{9}{1953125\cosh^9\left(\frac{x}{10}\right)\Gamma(\alpha+1)}\\
&\quad \times\left(1250t^\alpha\cosh^7\left(\frac{x}{10}\right) - 1731t^\alpha\cosh^5\left(\frac{x}{10}\right) + 36t^\alpha\cosh^3\left(\frac{x}{10}\right) + 63t^\alpha\cosh\left(\frac{x}{10}\right)\right.\\
&\quad \left. + \left(1250t^\alpha\cosh^6\left(\frac{x}{10}\right) - 769t^\alpha\cosh^4\left(\frac{x}{10}\right) - 108t^\alpha\cosh^2\left(\frac{x}{10}\right) + 9t^\alpha\right)\sinh\left(\frac{x}{10}\right)\right)\\
&\quad + \frac{9}{610351562500\cosh^{16}\left(\frac{x}{10}\right)\Gamma(2\alpha+1)}\left(18750000t^{2\alpha}\cosh^{14}\left(\frac{x}{10}\right)\right.\\
&\quad - 146475656t^{2\alpha}\cosh^{12}\left(\frac{x}{10}\right) + 190575082t^{2\alpha}\cosh^{10}\left(\frac{x}{10}\right) - 14558217t^{2\alpha}\cosh^8\left(\frac{x}{10}\right)\\
&\quad \left. - 42840306t^{2\alpha}\cosh^6\left(\frac{x}{10}\right) + 3233385t^{2\alpha}\cosh^4\left(\frac{x}{10}\right) + 385884t^{2\alpha}\cosh^2\left(\frac{x}{10}\right)\right.
\end{aligned}$$

$$+2\left(9375000t^{2\alpha}\cosh^{13}\left(\frac{x}{10}\right)-65824672t^{2\alpha}\cosh^{11}\left(\frac{x}{10}\right)+63206373t^{2\alpha}\cosh^{9}\left(\frac{x}{10}\right)\right.$$
$$+15972336t^{2\alpha}\cosh^{7}\left(\frac{x}{10}\right)-10377315t^{2\alpha}\cosh^{5}\left(\frac{x}{10}\right)-291600t^{2\alpha}\cosh^{3}\left(\frac{x}{10}\right)$$
$$\left.+44226t^{2\alpha}\cosh\left(\frac{x}{10}\right)\right)\sinh\left(\frac{x}{10}\right)-7290t^{2\alpha}\Big)+\ldots.$$

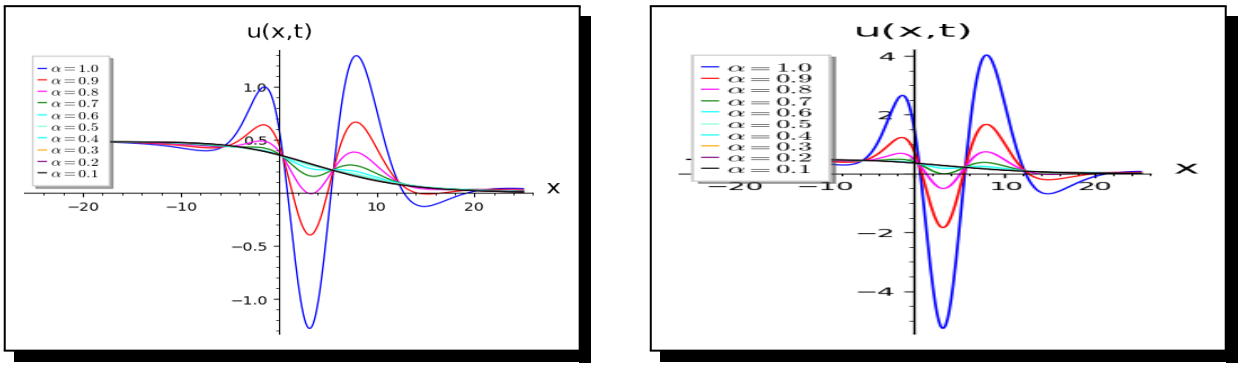


(a) $t = 10$ **(b)** $t = 20$

(c) $t = 30$

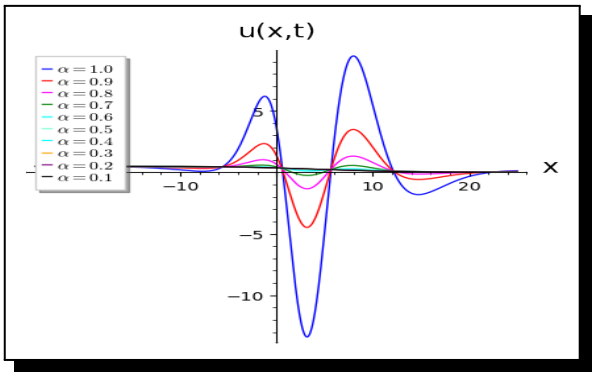
Figure 3. Solutions at different time levels $t \le 30$

From Figure 3, we observe the kink type travelling wave solution for $0 < \alpha \le 1$, $t \le 10$. Furthermore, we observe the variation in wave patterns for $0.9 \le \alpha \le 1$, $20 \le t \le 30$.



(a) $t = 50$ **(b)** $t = 75$

Figure 4 (*continued*)



(c) $t = 100$

Figure 4. Solutions at different time levels $t > 30$

In Figure 4, we observe that the behavior of the traveling waves follows a specific pattern as time increases in the range $50 \le t \le 100$.

Case 4: $n = 4$

The mKdV Burger's equation (1) will take the form as

$$\frac{\partial^\alpha u}{\partial t^\alpha} + u^4 \frac{\partial u}{\partial x} - a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^3 u}{\partial x^3} = 0, \quad 0 < \alpha \le 1. \tag{13}$$

Adomian polynomials for nonlinear term $u^4 \frac{\partial u}{\partial x}$ are given by

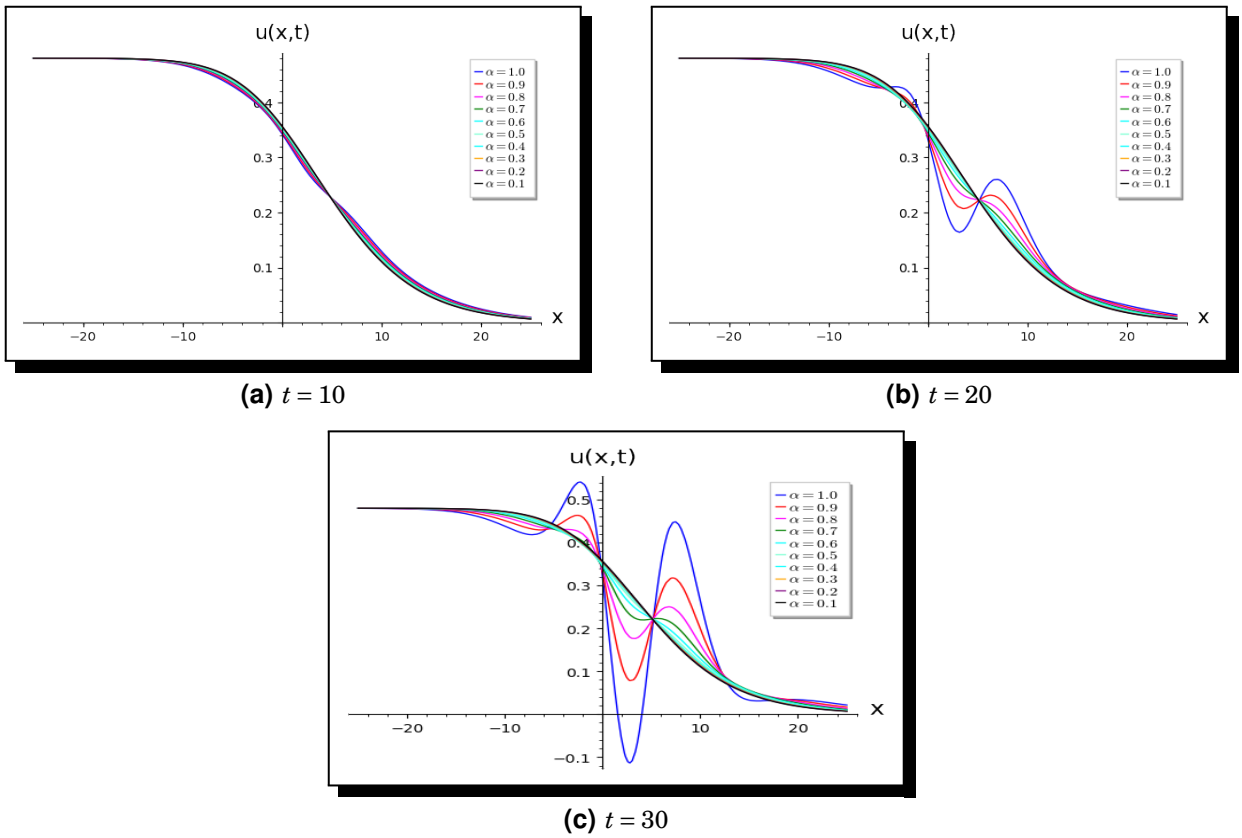
$$\begin{aligned}
A_0 &= u_0^4 u_{0x},\\
A_1 &= 4u_0^3 u_{0x} u_1 + u_0^4 u_{1x},\\
A_2 &= 6u_0^2 u_{0x} u_1^2 + 4u_0^3 u_1 u_{1x} + 4u_0^3 u_{0x} u_2 + u_0^4 u_{2x},\\
A_3 &= 4u_0 u_{0x} u_1^3 + 6u_0^2 u_1^2 u_{1x} + 12u_0^2 u_{0x} u_1 u_2 + 4u_0^3 u_{1x} u_2 + 4u_0^3 u_1 u_{2x} + 4u_0^3 u_{0x} u_3 + u_0^4 u_{3x},\\
&\vdots
\end{aligned}$$

Using similar procedure in *Case* $n = 2,3$, we obtain the series solution as

$$\begin{aligned}
u(x,t) = &\left\{\frac{3}{25}\frac{a^2}{b}\operatorname{sech}^2\left(\frac{a}{10b}x\right) + \frac{6}{25}\frac{a^2}{b}\left[1 - \tanh\left(\frac{a}{10b}x\right)\right]\right\}\\
&+ \frac{9}{48828125\cosh^{11}\left(\frac{x}{10}\right)\Gamma(\alpha+1)}\left(31250t^\alpha\cosh^9\left(\frac{x}{10}\right) - 45147t^\alpha\cosh^7\left(\frac{x}{10}\right)\right.\\
&+ 432t^\alpha\cosh^5\left(\frac{x}{10}\right) - 216t^\alpha\cosh^3\left(\frac{x}{10}\right) + 243t^\alpha\cosh\left(\frac{x}{10}\right)\\
&+ \left(31250t^\alpha\cosh^8\left(\frac{x}{10}\right) - 17353t^\alpha\cosh^6\left(\frac{x}{10}\right)\right.\\
&\left.\left. - 1296t^\alpha\cosh^4\left(\frac{x}{10}\right) - 648t^\alpha\cosh^2\left(\frac{x}{10}\right) + 27t^\alpha\right)\sinh\left(\frac{x}{10}\right)\right)\\
&+ \frac{9}{381469726562500\cosh^{20}\left(\frac{x}{10}\right)\Gamma(2\alpha+1)}\left(11718750000t^{2\alpha}\cosh^{18}\left(\frac{x}{10}\right)\right.\\
&\left. - 93397779464t^{2\alpha}\cosh^{16}\left(\frac{x}{10}\right) + 119340738058t^{2\alpha}\cosh^{14}\left(\frac{x}{10}\right)\right.
\end{aligned}$$

$$
\begin{aligned}
&-27220195257t^{2\alpha}\cosh^{12}\left(\frac{x}{10}\right)+6164358768t^{2\alpha}\cosh^{10}\left(\frac{x}{10}\right)\\
&-8880681672t^{2\alpha}\cosh^{8}\left(\frac{x}{10}\right)+357643512t^{2\alpha}\cosh^{6}\left(\frac{x}{10}\right)\\
&-22534848t^{2\alpha}\cosh^{4}\left(\frac{x}{10}\right)+8406828t^{2\alpha}\cosh^{2}\left(\frac{x}{10}\right)\\
&+2\Big(5859375000t^{2\alpha}\cosh^{17}\left(\frac{x}{10}\right)-40215172768t^{2\alpha}\cosh^{15}\left(\frac{x}{10}\right)\\
&+39850950837t^{2\alpha}\cosh^{13}\left(\frac{x}{10}\right)+432504576t^{2\alpha}\cosh^{11}\left(\frac{x}{10}\right)\\
&+4567999320t^{2\alpha}\cosh^{9}\left(\frac{x}{10}\right)-1609638480t^{2\alpha}\cosh^{7}\left(\frac{x}{10}\right)\\
&-20575296t^{2\alpha}\cosh^{5}\left(\frac{x}{10}\right)-11757312t^{2\alpha}\cosh^{3}\left(\frac{x}{10}\right)\\
&+669222t^{2\alpha}\cosh\left(\frac{x}{10}\right)\Big)\sinh\left(\frac{x}{10}\right)-83106t^{2\alpha}\Big)+\cdots
\end{aligned}
$$

From Figure 5, we observe the kink type travelling wave solution for $0 < \alpha \le 1$, $t \le 10$. Furthermore we observe the variation in wave patterns for $0.9 \le \alpha \le 1$, $20 \le t \le 30$.

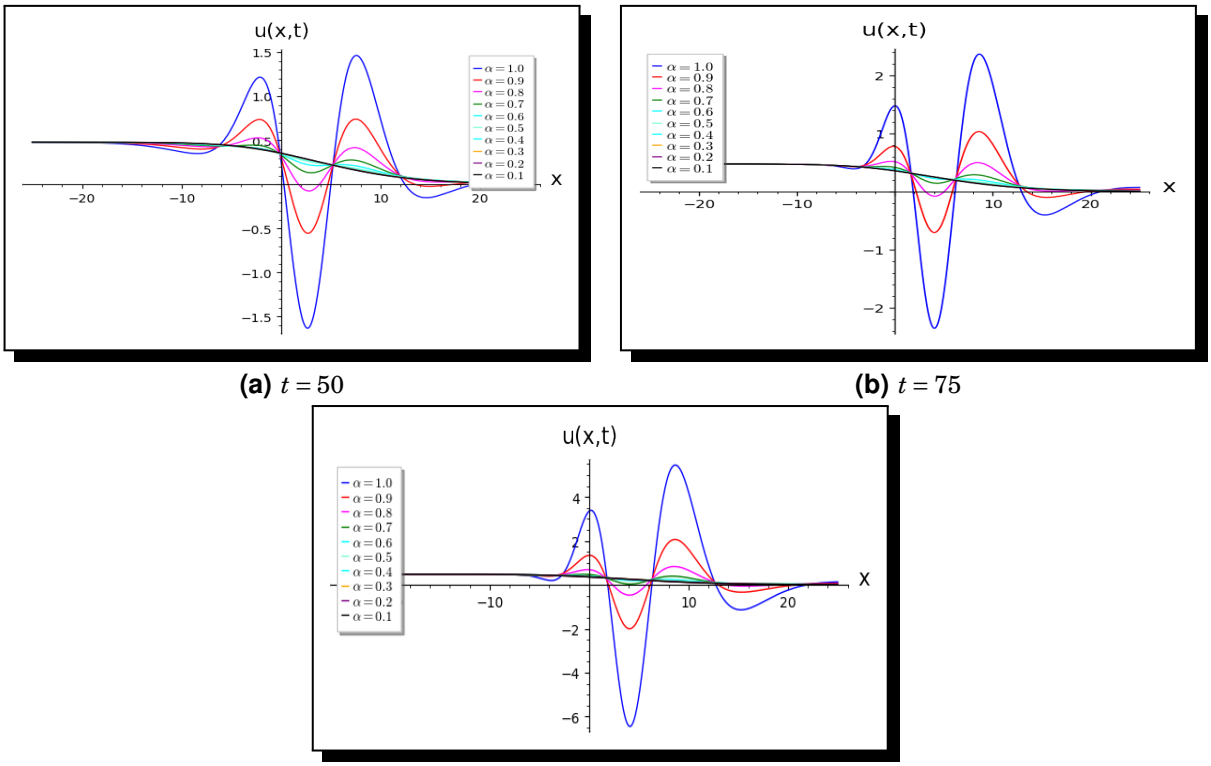


(a) $t = 10$ **(b)** $t = 20$

(c) $t = 30$

Figure 5. Solution profiles for varying $t \le 30$

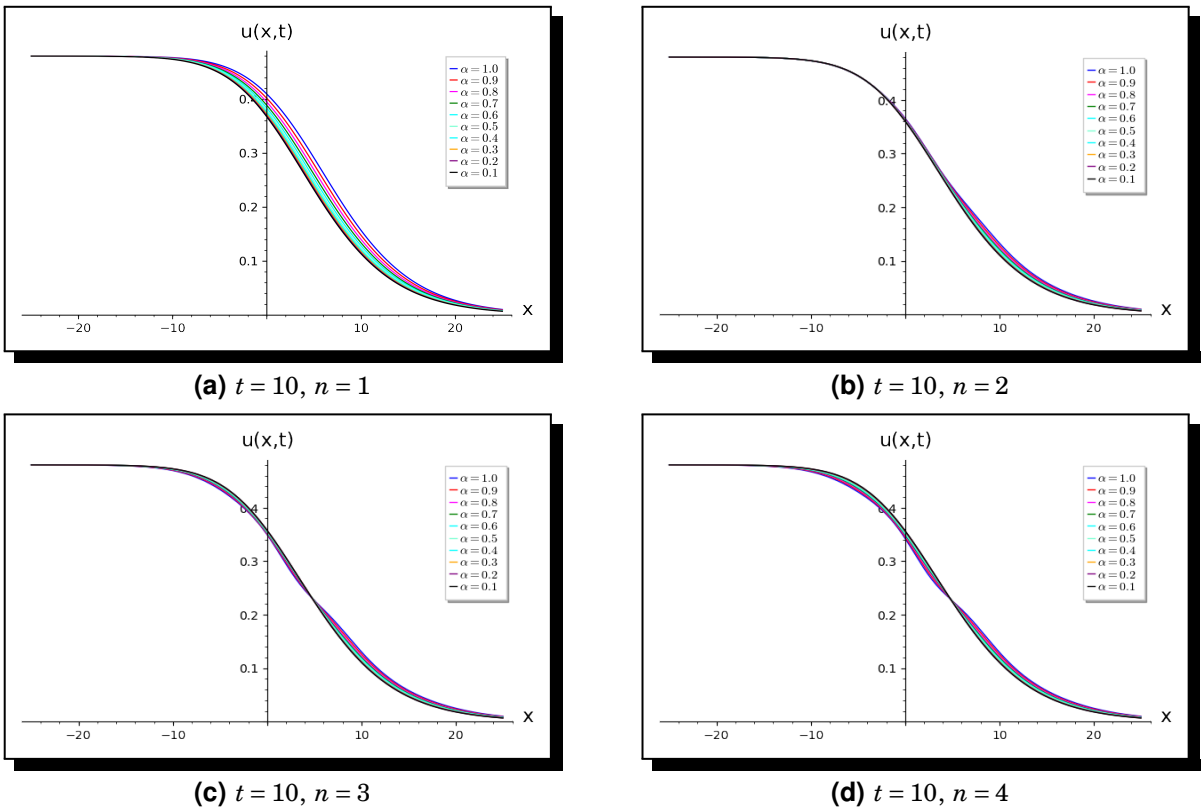
In Figure 6, we observe that the behavior of the traveling waves follows a specific pattern as time increases in the range $50 \le t \le 100$.



(a) $t = 50$ **(b)** $t = 75$

(c) $t = 100$

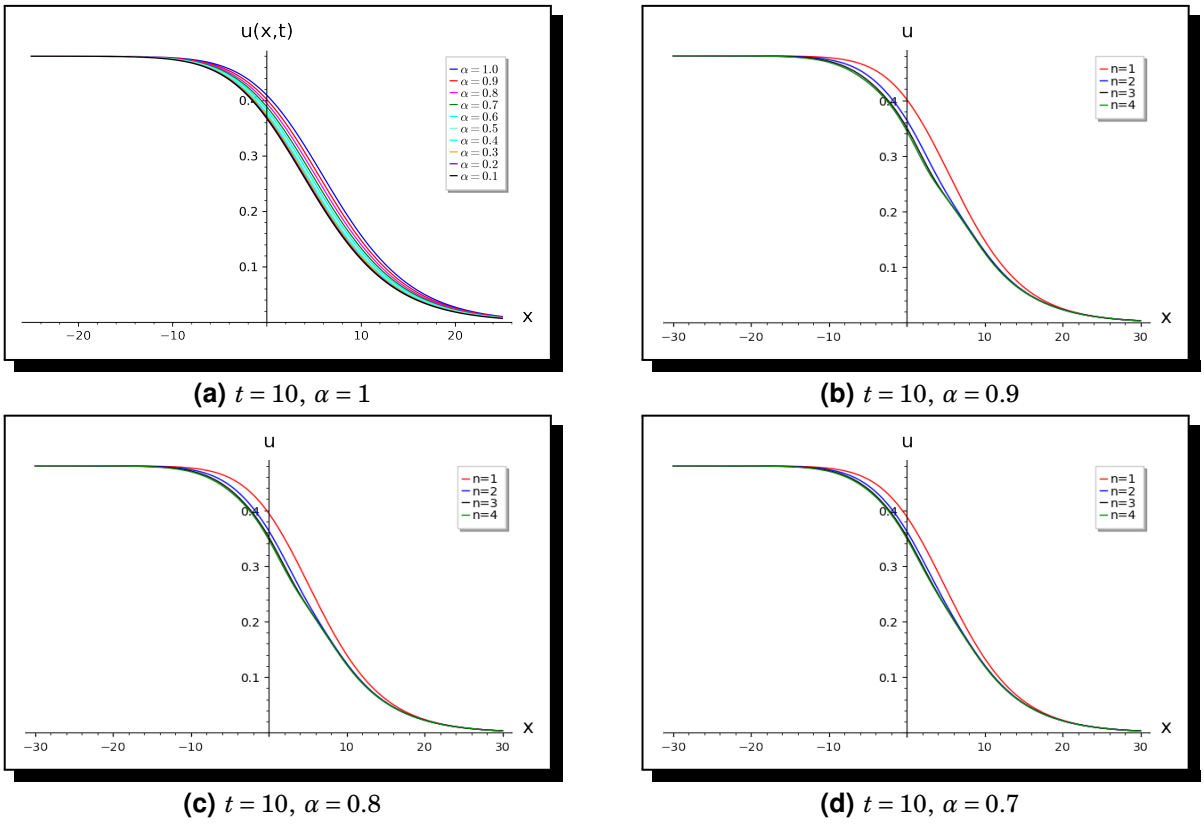
Figure 6. Solution profiles for varying $t > 30$



(a) $t = 10,\ n = 1$ **(b)** $t = 10,\ n = 2$

(c) $t = 10,\ n = 3$ **(d)** $t = 10,\ n = 4$

Figure 7. Variation of the solution curves with n at $t = 10$



(a) $t = 10,\ \alpha = 1$ **(b)** $t = 10,\ \alpha = 0.9$

(c) $t = 10,\ \alpha = 0.8$ **(d)** $t = 10,\ \alpha = 0.7$

Figure 8. Effect of α on the solution at $t = 10$

3. Conclusion

We successfully applied the ADM to solve the time-fractional modified KdV Burger's equation. We observed that for $n = 1$, the obtained results and the exact results of the proposed problem are closely related. The convergence related to integer order results gets confirmed through the graphical representation. For short memory period $0 < t \leq 10$, we observe the fast convergence, and the obtained solutions are closer to the exact solution. For the memory period $10 < t \leq 20$, variation in the wave patterns is observed for $0.7 \leq \alpha \leq 1$. For long memory period $t > 30$, we observe specific pattern formation in the behavior of traveling wave solutions for $\alpha = 1, 0.9, 0.8, 0.7$. From the graphical representation, we observe that as nonlinearity increases for $n = 2, 3, 4$, time fractional modified KdV Burger's equation preserves the kink-type solutions.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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